Eur. Phys. J. Special Topics **225**, 2453–2464 (2016)

DOI: 10.1140/epjst/e2016-60023-1

THE EUROPEAN PHYSICAL JOURNAL SPECIAL TOPICS

Review

Soliton molecules for advanced optical telecommunications

Fedor Mitschke[a], Alexander Hause, and Christoph Mahnke

Universität Rostock, Institut für Physik, 18059 Rostock, Germany

Received 10 February 2016 / Received in final form 5 July 2016
Published online 22 November 2016

Abstract. Recent developments in the technology of optical telecommunications are pushed forward by the rapidly growing demand for data-carrying capacity. Current approaches are discussed; most lines of investigation are limited to the linear (i.e. low power) regime. It is shown how this restriction poses a limit for further evolution. If, on the other hand, the nonlinear regime is entered, recent developments about soliton molecules offer a possibility to advance further.

1 State of the art in optical telecommunications, and a dilemma

The current technology of fiber-optic data transmission is enormously powerful. Rapid streams of short (picosecond) light pulses pass through optical fibers; data is encoded onto the pulse sequence. In this way, fibers today carry the bulk of all transmitted information, from voice signals of conventional telephone to the data packets of internet traffic. This technology is so successful because fibers have very low power loss over an extremely wide bandwidth, on the order of a few tens of terahertz. It represents a challenge that the demand for the volume of data to be transmitted grows relentlessly: After voice, email, and stationary web sites came more data-hungry services like streaming video. It is not anticipated that the growth will abate any time soon; indeed, the proposed internet connection of all kinds of appliances and objects (the internet of things) will cause another surge. Note that even wireless devices rely on the fiber-optic network backbone.

Thanks to the enormous capabilities of optical fibers, supply of data-carrying capacity has, by and large, kept up with demand so far. In the face of a continued exponential growth of demand, however, fiber technology as we know it is about to reach its limit [1], and there is talk of a "capacity crunch" [2]. The entire available bandwidth for low-loss transmission is now being used. Also, in addition to amplitude modulation, phase and polarization of light signals begin to be used for coding data.

2 Bandwidth and coding

It is instructive to revisit Claude Shannons celebrated theorem about data-carrying capacity which remains a useful benchmark [3] and starting point for further

[a] e-mail: fedor.mitschke@uni-rostock.de

discussion. It defines the channel capacity C (the maximum data rate for error-free transmission). To suit our present context we rewrite it as

$$C = B \log_2(k). \tag{1}$$

Here, B is the available bandwidth, which for optical fibers can be estimated at 30–50 THz depending on how much loss one deems acceptable. It is not to be expected that this figure can be substantially improved by any available means. The other factor is the binary logarithm of k which stands for the number of symbols distinguished in the coding. For the analog systems considered by Shannon the argument of the log is $1 + S/N$, with S and N the signal power and noise power; respectively. This expression is an upper bound for k which is not always exhausted; e.g. in binary coding with $S/N \gg 1$.

In the lion's share of all transmission systems in operation today, there is binary coding: the absence of light within a clock period represents a logical *Zero*, and a light pulse stands for a logical *One*. This is known as 'on-off keying', a binary amplitude modulation for which $k = 2$.

In the 1990s *wavelength division multiplexing* (WDM) was introduced to fiber optics in order to make use of the fibers entire low loss window. Of course, no single data source can ever provide a data rate of several terabits per second. Therefore, data streams from many separate sources are combined into subchannels, each with a data rate which can be handled with electronic means. Each subchannel is encoded onto a different optical frequency (the carrier); these are then combined into a single data stream with optical means. This technique allows to come close to the Shannon limit for binary coding using up to several hundreds of subchannels, each of which carries e.g. 40 or 100 GBits/s.

With B pushed to its limit, the only way to further increase C is through increasing k: the coding must be optimized. To this end, the use of light pulses with different peak power, polarization and phase was considered. Two orthogonal states of polarization provide an extra factor-of-two (one more bit) of information per clock period provided that the cross talk between the orthogonal states is kept in check. Then, techniques from traditional radio engineering were adopted for further enhancement in recent years. For example, in quaternary phase shift keying (QPSK) four symbols are used which differ in optical phase but have the same amplitude; this was designed to minimize possible interference from nonlinearity which acts on amplitude or power. This also provides one extra bit per clock period. Then, a combination of phase and amplitude modulation known as QAM (quadrature amplitude modulation) was introduced, and QAM has been demonstrated to support up to 11 bits of information per clock period [4].

Shannon, who actually considered pure analog amplitude modulation of a radio frequency carrier, stated clearly that the number of distinguishable amplitude values is limited by the noise present at the detector. This insight carries over, of course, to other modulation formats because noise is unavoidable and ubiquitous, and is present in both amplitude and phase.

The organization of the various transmission symbols is often displayed in a 'configuration space' representation, this configuration space has more than one dimension as soon as degrees of freedom beyond amplitude are used. For symbols with different phase and amplitude values one uses the complex plane in which radius corresponds to amplitude, and angle to phase angle. Each symbol is localized at a certain point in that plane. A well-designed QPSK or QAM scheme will place symbols in an equidistant pattern, with the origin at the center of the set.

Of course, each symbol is subject to noise, and this is indicated in Fig. 1 by rendering the individual symbols as somewhat balls: the grey shading represents the noise.

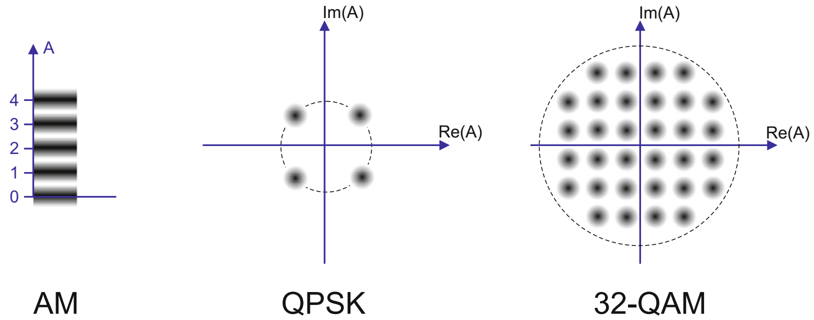

Fig. 1. Configuration space for three coding formats. Left: In conventional amplitude modulation, amplitude levels (real-valued) can only be distinguished when they are spaced by at least as much as is dictated by the noise that blurs them. Center: In quadrature phase shift keying, the configuration space is the plane of the complex amplitude. Four symbols are placed at positions of equal amplitude, at the largest distance from each other compatible with that. Right: This example of a quadrature amplitude modulation with 32 symbols (32-QAM) codes 5 bits into one clock cycle; the picture is inspired by [5]. The symbols are placed in a regular pattern; each carries a blur due to noise. A certain maximum amplitude (dashed circle) is not exceeded.

The obvious conclusion is that in the presence of noise symbols must be chosen with sufficient intersymbol spacing, to avoid their overlap and the resulting transmission errors.

3 The importance of taking nonlinearity into account

Transmission of light pulses through optical fiber is fundamentally different from transmission of light signals through the atmosphere, radio signals through cables, or through open space: While in all other cases the transmission medium may be thought of as linear, fiber is inherently nonlinear. The glass of which the fiber is made has an intensity-dependent response to the light field travelling in it, with a variation of refractive index caused by the light known as the optical Kerr effect. As a result there is self phase modulation in each light pulse, and four wave mixing and cross phase modulation between light signals of different frequencies. There are more nonlinear effects like Raman shift to which we will return below.

In a linear medium, the superposition principle holds: different parts of the compound data stream do not affect each other. In a nonlinear system, mixing products create interchannel cross talk and subsequently bit errors at the detector. For radio transmission in the atmosphere, mixing products between different channels occur only under some extreme and rare ionospheric circumstances, but in fibers they arise naturally. On the bright side, they are at least deterministic and therefore in principle predictable – provided that sufficient information is available. In any event, concepts from other areas of telecommunication cannot be blindly transferred to optical fiber; the impact of nonlinearity must be understood.

Corrections to Shannons equation (Eq. (1)) due to nonlinearity have been given [6,7]. Typically, nonlinearity is detrimental to the obtainable data-carrying capacity. The reason is that for more capacity, more symbols are added to the configuration as in Fig. 1 (right). Due to noise considerations, they can only be added on the outside of the cloud, i.e. at larger amplitudes. The higher powers thus involved increase the interchannel cross-talk due to mixing processes. It may be worthwhile to point out, however, that a recent paper showed how in certain situations nonlinearity may also be put to an advantage so that an improvement over the Shannon limit becomes possible [8]. The central idea of that suggestion is that as the noise builds up gradually during transmission, nonlinearity can be employed to repeatedly collapse each noise ball to its center point; then symbols can be packed more densely in configuration space so that the capacity is increased.

In technical systems, it is standard practice to keep the power quite low to avoid the impact of nonlinear effects. Considering the power loss over the transmission distance, one must then deal with signal-to-noise ratio issues at the detector. When adding more symbols to a QAM scheme, there are has only two choices: Either to append them on the outside, by which one accepts larger amplitudes. Or by keeping the maximum amplitude bounded and by reducing the intersymbol distance, by which one increases the configuration space density. In the former case, there will be more transmission errors due to nonlinearity-induced crosstalk, in the latter more errors will arise from noise interference.

This is a fundamental dilemma. In many recent research papers ever more ambitious multibit coding schemes are demonstrated [9]. In these cases usually researchers follow one of the two available options to work around the dilemma, or a combination thereof. Either they restrict their system to shorter reach because in a shorter fiber there is less loss. Loss shrinks the symbol spacing in configuration space while the noise balls basically remain the same. Therefore, a shorter link length leaves some margin to pack the symbols closer. Or they accept the arising transmission errors, and mitigate them through software-based error correction. In some cases, it was accepted that the bit error rate reached 10^{-3} where the telecom industry for a long time required 10^{-8} to 10^{-12} for standard transmission protocols. Through refined algorithms, errors can be detected and corrected, at a slight expense of a certain overhead in transmitted data. It is not difficult to correct isolated bit errors, but as their rate increases, the number of errors of several subsequent bits increases sharply, and that is harder to correct. In any event, it appears prudent and good engineering practice to demand that the physical layer, by itself, has a certain reliability and dependability.

Highly respectable results in terms of capacity have been obtained along these lines by using sophisticated coding schemes combined with complex error compensation techniques (for a recent review see [10]). But ultimately the available number of symbols is limited to the available configuration space area, divided by the 'noise ball' area assigned to each symbol. As this limit is approached, basically one returns to an analog system. This is peculiar as it was precisely the robustness of binary systems that motivated the abandonment of linear systems many years ago. One concludes that by going beyond plain on-off keying one either has shorter reach, or higher bit error rate, or a combination thereof. What one gains in capacity, one loses in reach or data integrity.

4 How to escape the dilemma: What alternatives are there?

This compromising is unsatisfactory. A few possibilities remain:

More fibers. Of course one might simply deploy more fibers. This is already done at an astonishing rate: many millions of kilometers per year correspond, computationally, to supersonic speed. But this approach is not only expensive. It is ultimately futile because when basically the same technology is maintained, one important metric is preserved: the cost per bit remains essentially constant. As exponential traffic growth requires corresponding deployment of additional fibers, it follows that there will be exponentially growing cost. Beyond the financial side, also consider the energy consumption which is subject to the same growth and ultimately represents a major cost factor, and an environmental issue to boot [11]. Therefore, in the long run this brute force 'more of the same' approach is not viable.

Nonlinearity mitigation. Currently there is ongoing research that exploits the predictability of nonlinear impairment in fibers. Mathematical models exist that are quite accurate in describing fiber propagation, with all nonlinear effects included. The idea is to detect the signal at the distal fiber end, however distorted it may be, and subject it to backward propagation – preferably not in a second real fiber, of course but in a computer simulation [12]. In an ideal case all distortions – linear (dispersion) as well as nonlinear – could be undone, but in practice a perfect cancellation is not possible, and success has been modest so far [13]. Also, the computation time adds considerably to the transmission latency – and this at a time where a new fiber is being laid through Arctic waters with the express purpose to cut down on latency [14]: By the unconventional geographic route, the distance between financial centers is reduced, and that allows to shave off several milliseconds from the transmission time. In these days of high-frequency trading at stock exchanges, apparently milliseconds can make or break fortunes.

Spatial multiplexing. Currently, approaches are hotly debated which exploit the transverse spatial dimensions, i.e. the location of the power transport within the cross section of the fiber. These concepts forgo the time-proven single-mode fiber with its well-defined guiding mechanism. After a suggestion in [15], two related approaches have been pursued in recent years:

- Mode divison multiplexing (MDM) uses several modes of a conventional multi-mode fiber [16].
- Space division multiplexing (SDM) uses specialty fibers with several cores side by side, distributed over the cross section [17].

Terminology has not been quite uniform from the beginning, and occasionally SDM was used as a generic term for both formats. Sometimes several single- and multimode cores are combined into one fiber, effectively creating a mixed MDM / SDM arrangement [18]. A variant is the use of hollow-core fibers: Wide hollow cores promise to guide several modes at minimal loss and nonlinearity because the overlap area between field and glass is small. For the same reason the effective index is close to unity; these fibers are therefore advertised as guiding light 'at the speed of light in vacuum' [19], a feature which allows reduction of latency (see previous item).

The central issue to be addressed in these formats is cross talk between data streams. In MDM there is the natural coupling between modes which are orthogonal only in mathematical abstraction; in reality any minimal deformation of the fiber readily causes coupling of light into other modes. The question is not for how far the modes can be maintained as such – that would not be very far –, it is over which distance the *orthogonality* can be preserved. That gives more hope, but still, long haul (transoceanic) is not likely to work in this scheme. In SDM the number of cores is limited by crosstalk considerations due to proximity of cores and long interaction lengths. A new, larger fiber diameter of $\approx 225\ \mu$m to accomodate several cores has been suggested to replace the conventional 125 μm fiber [20]. At that diameter the record seems to stand at 30 cores [21]; for an even thicker fiber with heterogenous (unequal index) cores, at 36 [22].

The major objection against all spatial multiplexing schemes is that legacy fiber cannot be used. A vast network of single-mode fibers is in place and covers 'the four corners of the world'. It represents a staggering financial investment somewhere in the 'teradollar' range, considering that the installation of a single transatlantic link costs on the order of 1 billion US $. This tremendous asset of installed cables is useless for MDM/SDM formats, while it remains unclear whether major financial and energetic savings are possible in comparison to several conventional fibers run in parallel. Economic considerations definitely favor the continued use of existing (legacy) fibers.

5 What about solitons?

From a physics point of view, there is one more option, possibly an elegant way to get out of the dilemma. Under the right circumstances, nonlinear waves can self-stabilize, i.e. form localized structures in space and/or time. The central concept in this context is the *soliton*, or solitary wave. One particular embodiment of solitons is the fiber-optic soliton pulse, a particular type of light pulse in an optical fiber with remarkable properties. It was predicted in 1973 [23] and first demonstrated in 1980 [24]. Solitons rely on a balance between the nonlinearity-induced phase modulation and linear dispersive effects. They are stable solutions of the underlying nonlinear propagation equation, known as the nonlinear Schrödinger equation (NLSE) [25–27], Their stability implies robustness in the presence of perturbations: perturbations can heal out (in reality, to some degree). Therefore solitons have been considered by many to be the natural bits for optical telecommunications.

After a lot of research into detail like behaviour under amplification, upon collision, etc., this scheme has eventually found its way into industrial application: In the first years of the 21st century, solitons had advanced from a lab curiosity to commercial reality in optical telecommunications.

Soliton transmission has the undisputed advantage that it takes nonlinearity into account from the outset. A consequence of the somewhat higher power is that noise issues are not nearly as severe, and the reach is not compromised. However, solitons – being entities that rely on nonlinearity – cannot simply be scaled in amplitude to yield more symbols, as required for QAM. For this reason they were always considered by the vast majority of researchers to be useable in on-off keying only. When the necessity arose to study beyond-binary coding schemes, solitons more or less fell out of grace. However, this conclusion was premature: Enhanced coding with compound solitonic structures was first suggested in [28], and [29] considers phase keying of similar entities; phase keying of standard solitons was recently discussed in [30]. Here we consider the concept of *soliton molecules*.

6 The soliton molecule concept

Soliton molecules are stable compounds of solitons. However, before discussing them we first need to point out that the preferred type of fibers is now the so-called dispersion-managed fiber (DM fiber).

DM fibers are fibers built from periodically alternating segments with positive and negative group velocity dispersion coefficients. This brings the path average dispersion to a small value; this was initially intended to suppress dispersive affects. It also creates phase mismatch for the four wave mixing process and thus thwarts interchannel mixing products. Soliton-like pulses, in the sense of pulses that stabilize themselves due to a balance between dispersive and nonlinear effects, exist in

DM fibers [31–35] and are commonly called DM solitons. Their shape resembles a Gaussian more closely than that of conventional solitons [36–39], and the pulse shape 'breathes' over the period of the dispersion structure. However, the shape periodically returns to the initial one, so that one can call it *in the stroboscopic sense* (i.e. when sampled once per dispersion period).

It is well known that depending on the relative phase of solitons in a fiber there can be a net attraction or repulsion [40,41]. Equal-energy soliton pairs with constant temporal separation therefore do not exist in constant-dispersion fibers but see [28]). In DM fibers the situation is different, as is clear from the experimental demonstration that stable bound states of equal-energy DM solitons exist [42]. The same was also noted in theoretical work [43–46], and was recently pursued in [47,48]. The pairs have a favored temporal separation which is characterized by a stable equilibrium of attracting and repelling forces. This is reminiscent of the equilibrium spatial separation of two nuclei in a diatomic molecule; we therefore call this structure a *soliton molecule*. The binding mechanism is based on the interaction forces which, however, are strongly modified in DM fibers; this was shown in detail in [49]. It remained open in Ref. [42] whether soliton molecules of more than two solitons exist, but there had been numerical indications [44,45,48,50].

Two-soliton compounds in constant-dispersion fibers, rather than in the now commonly deployed DM fibers, are treated in recent theoretical work in [51,52]; these are of course qualitatively different from the DM soliton molecules discussed here. Recent work on the latter deals with their stability [53], multiple equilibrium separations [54], and their average dynamics [55].

7 Experimental setup

A first demonstration of soliton molecules in [42] was restricted to a quite short fiber, and to two-soliton molecules. Later, an improved version [56] included a longer fiber, and at the same time introduced a new detection technique. An improved algorithm ('very advanced method of phase and intensity retrieval of E-fields', or VAMPIRE) was employed to obtain amplitude and phase information; it was later shown that in contrast to the more common 'frequency-resolved optical gating', or FROG technique [57] it works even in the case of spectra with nulls [58]. The binding mechanism of the solitons was elucidated through a perturbative study in [49].

The current state of the demonstration experiment represents another vast improvement, and also allows to create three-soliton molecules. It is shown schematically in Fig. 2; for more detail see [59,60]. A modelocked optical parametric oscillator (OPO), pumped by a Ti:sapphire laser, provides a stream of pulses at a wavelength tunable between 1480 nm and 1600 nm but mostly set to $\lambda_0 = 1540$ nm. Through a modification we obtain a pulse duration of $T_0 = 150\,\text{fs}$ corresponding to a full width at half maximum of $\tau_{\text{FWHM}} \approx 250$ fs, with a repetition rate of $\nu_{\text{rep}} = 57$ MHz. The modification was required for correct scaling of the experiment in comparison to a real-world application. The nearly Gaussian pulses can be modeled well by a field envelope

$$u(t) = \sqrt{P_0}\,\exp\left[-\frac{1+iC}{2}\,\frac{t^2}{T_0^2}\right] \tag{2}$$

with peak power P_0 (typically $P_0 = 30$ W) and chirp parameter $C \approx 0.4$.

We steer these pulse into a pulse shaper consisting of a spatial light modulator (SLM) in a symmetric $4-f$ setup with two gratings and two lenses [61]. The SLM allows amplitude and phase modulation in the spectral domain, within the limits of resolution set by its 128 pixels. This way we can carve single, double or triplet pulse

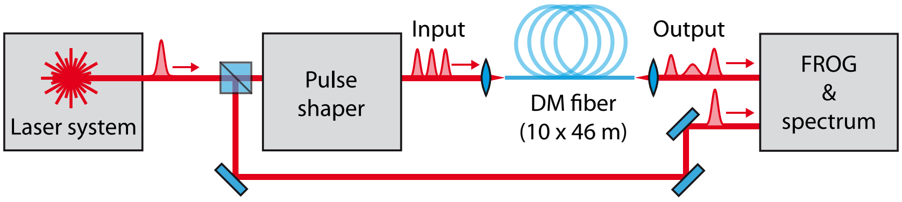

Fig. 2. Experimental setup.

shapes with – within bounds – arbitrary phase profiles from the OPO pulses, and also cancel its chirp in the process.

To model pulse pairs, the addition of two Gaussians in antiphase, at a center-to-center separation of $\sigma_{\rm in}$ on the order of twice the pulse width, is a good starting point. Similarly, pulse triplets can be formed from the addition of three Gaussians with alternating phase at equal separations. We address the SLM with the Fourier transform of such condition, suitably pixellated, to generate the targeted structure. We note in passing that one can do much better than presupposing Gaussian shapes: Propagation of signals in the fiber can itself reveal the pulse shapes for which best fidelity of pulse shape preservation is obtained; the optimized shape can be determined through a search iteration employing a genetic algorithm [62].

The structure so prepared is launched into a length of DM fiber, having 10 dispersion periods of alternating anomalously and normally dispersive fiber. The fiber line was designed to represent a scale model of a typical commercial fiber line with 40 GBit/s bit rate using $\tau_{\rm FWHM} = 7.5$ ps pulses. Scaling down pulse duration by some factor implies scaling down the fiber length by the square of that factor. In our layout, length is therefore scaled down 900-fold, and our ten dispersion periods (460 m of fiber) correspond to a real-world fiber length of ca. 400 km. We can afford to pay the price of an upscaling of the required power level to ≈ 10 pJ for a soliton. Higher-order perturbations like higher-order dispersion, Raman shift, etc. [25,27] scale with higher powers of the scaling factor and are thus much more dominant in our setup then they would be in a real-world setting. In that sense, our experiment makes conservatively cautious predictions about their impact. Splice losses dominate the loss budget; they were most carefully assessed.

Data acquisition uses FROG in order to obtain both amplitude and phase of the received signals. In its Blind FROG version, cross correlation of the pulse under test with a reference straight from the OPO is evaluated. All experiments were accompanied with numerical simulations (using split-step Fourier algorithm [25]) taking all higher-order corrections, splice losses, etc. fully into account.

8 Experimental results

By suitable programming of the SLM, we could generate and successfully transmit both two-soliton and three-soliton molecules. Systematic tests provided information about parameter ranges of their existence. A relevant quantity is the separation between adjacent pulses in the launch condition $\sigma_{\rm in}$, as stable molecules are only created in a certain range of this separation. The hallmark of a stable structure is that it arrives unchanged at the distal fiber end, in particular with pulse separation σ the same as $\sigma_{\rm in}$.

It is clear that with a given fiber link, signals can only be intercepted and evaluated at a particular propagation distance, i.e. at the end of the fiber. However, we obtain additional information about the evolution of shapes through evaluation of the

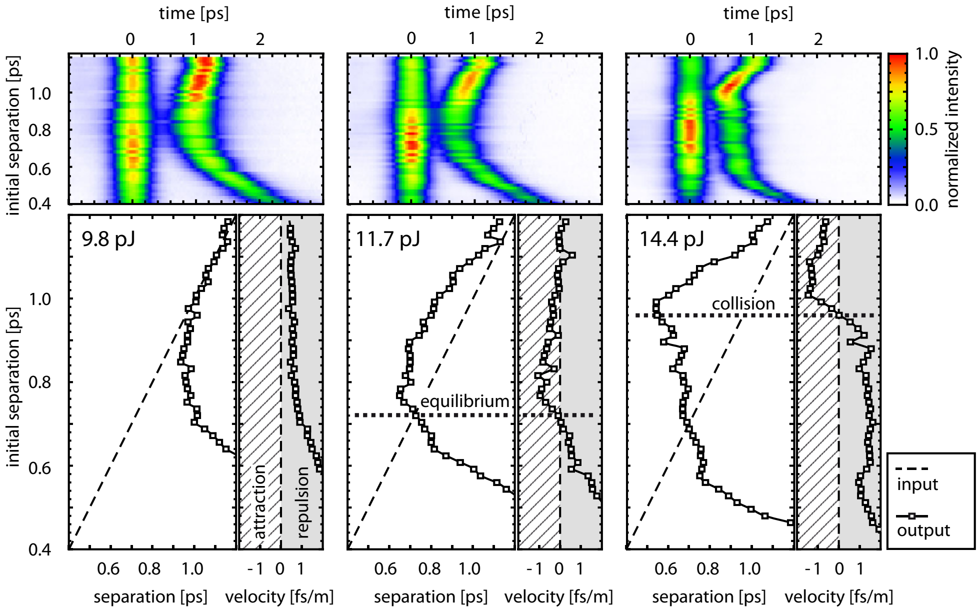

Fig. 3. Evolution of separation between pulses in a pair when the initial separation is varied. The three columns pertain to three different launch energies as indicated (numbers refer to for single-pulse energy). Top: measured cross-correlation traces, color coded. The left pulse was centered on $t = 0$ to remove timing fluctuations. Below: Measured separation and velocity (data points) compared to initial separation (dashed lines). Stable equilibrium $\sigma_{\rm eq} \approx 0.72$ ps is characterized by separation being equal to input separation, and simultaneously velocity equal to zero.

phases, as provided by the FROG technique. Note that a uniform phase tilt across the structure amounts to a frequency shift. That, in turn, translates to a shifted velocity in a dispersive medium. We can therefore distinguish the parameter combinations of a stable propagation (where interpulse separation is maintained) from those where the initial separation is repeated at the fiber end by sheer coincidence while actually the pulses keep moving relative to each other.

To illustrate our systematic studies, it may suffice here to show an experiment in which the input separation for a two-soliton molecule is varied and the output separation monitored. Experimental data shown in Fig. 3 were obtained at three energy levels, increasing from left to right. From the original cross correlation data (top) the separation is extracted and shown in the left part of the lower panels, with the pertaining velocity adjacent to it. The dashed lines repeat the initial separation (initial velocity is zero throughout); intersection points of data with these lines indicate the absence of net change. In the low energy case there is no equilibrium, and for large separations the interaction becomes negligibly weak. In the highest energy case one sees a collision near 0.95 ps (close distance and velocity zero), but no equilibrium. On the other hand, in the intermediate case there is an unchanged separation and simultaneously a zero velocity at $\sigma_{\rm in} = 0.72$ ps. The sign of velocities above and below indicates that this equilibrium is indeed a stable one.

Figure 4 wraps up the central result. Each structure – single solitons, two-soliton molecules, and three-soliton molecules – have been transmitted through the same fiber. They were found to be reasonably stable; it is worthwhile to compare the fiber input signals to the output signals. Figure 4 shows that the amplitudes are all lower at

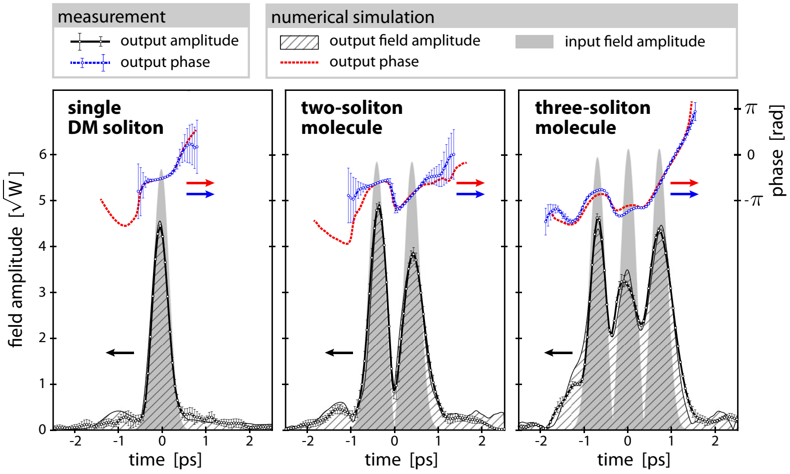

Fig. 4. Three different symbols: a single DM soliton (a), a two-soliton molecule (b), and three-soliton molecule (c). Field amplitudes of simulated input signals (grey area), same of the simulated (hatched area) and measured (black solid curve) output signal. Phases from simulation are shown as red line; the corresponding experimental traces (blue) have error bars which represent scatter from repeated FROG reconstructions.

the output, compared to the input: this is on account of fiber loss. Then, in the cases of the molecules, the peaks are of unequal height. This can be traced to the effect that power is coupled back and forth between participating pulses. The imbalance thus does not grow monotonously with distance; indeed the coupling much rather reverses direction at some point. Note, however, that in all cases the simulation matches the observed data very well so that we may conclude that all effects are suitably taken into account.

9 Conclusions

We have shown that a single soliton, a two-soliton molecule, and a three-soliton molecule all propagate stably in the DM fiber. Considering the absence of light in a clock period as another possibility, this completes a four-symbol alphabet which can encode 2 bits in one time step.

We have demonstrated the existence of soliton molecules in a proof-of-principle experiment, and determined conditions for their stability. So far we intentionally did not use amplifiers so that their own gain dynamics could not contribute. As a consequence we had to accept that pulse shapes got affected by the loss. Recently we have incorporated an Erbium-based amplifier at fiber midspan; results will be reported elsewhere. Also, as soon as wavelength division multiplexing is considered, the issue of collisions between pulses in different wavelength channels arises. This issue has been extensively studied for conventional and DM solitons [26]; we have now started to analyze the collision behaviour of soliton molecules and, again, will report results elsewhere.

A realistic transmission test with a random sequence of all symbols was not intended with the present setup. For such a test one would have to avoid bulky lab equipment, slow reprogramming of an SLM, and time-consuming data evaluation. We suggest that in a technical system all symbols would be simultaneously generated in synchronism by several diode lasers and accompanying microoptic circuitry, and only the symbol desired in a particular clock cycle would be routed to the fiber. Electro-optic modulators with sufficient speed to switch within the clock cycle are readily available. At the receiver, only the energy of each symbol needs to be assessed: We have convinced ourselves that the two-soliton molecule carries very nearly twice the energy of the individual soliton, and similarly for the three-soliton pulse, three times as much. Photodetectors with thresholding cicuitry are being used in QAM already and are readily available.

We have demonstrated a factor-of-two increase of the data rate, which admittedly is just a minor improvement. However, soliton molecules have well-defined phase and state of polarization so that it appears natural to combine the scheme proposed here with both polarization and phase multiplexing (in a similar spirit as in [30]), to make use of some of the best suggestions already in circulation. Of course, a combination with amplitude modulation is not feasible. Our scheme therefore does not reach the full amount of capacity increase presently demonstrated for QAM.

On the other hand we reiterate that our scheme fully takes nonlinearity into account and can therefore operate at somewhat higher power than linear schemes. This is an andvantage for the long haul where the soliton molecule scheme holds the promise of much longer distance to the next electronic regenerator. This scheme is also fully compatible with existing fibers; such advantages may well more than offset the disadvantage of slightly reduced capacity. For high-volume, short distance links (e.g. between data centers or between twin cities) linear schemes may well be suited best. Maybe different schemes will be used for different purposes in the future, in recognition of the fact that a nonlinear channel is quite different from a linear channel.

References

1. A. Chraplyvy, Plenary at European Conference on Optical Communications (ECOC) (Vienna, Austria, 2009)
2. D.J. Richardson, Science **30**, 329 (2010)
3. C.E. Shannon, Bell Syst. Tech. J. **27**, 379 (1948)
4. S. Beppu, K. Kasai, M. Yoshida, M. Nakazawa, Opt. Express **23**, 4960 (2015)
5. A. Sano, et al., Opt. Express **21**, 16777 (2013)
6. J.M. Kahn, K.-P. Ho, Nature **411**, 1007 (2001)
7. P.P. Mitra, J.B. Stark, Nature **411**, 1027 (2001)
8. K.S. Turitsyn, S.K. Turitsyn, Opt. Lett. **37**, 3600 (2012)
9. B.J. Puttnam, R.S. Luís, W. Klaus, J. Sakaguchi, J.-M. Delgado Mendinueta, Y. Awaji, N. Wada, Y. Tamura, T. Hayashi, M. Hirano, J. Marciante, paper PDP.3.1 in: *Proc. Eur. Conf. Opt. Commun. (ECOC) Valencia* (2015)
10. G. Li, Adv. Opt. Photonics **1**, 279 (2009)
11. W. Vereecken, W. Van Heddeghem, M. Deruyck, B. Puype, B. Lannoo, W. Joseph, D. Colle, L. Martens, P. Demeester, IEEE Commun. Mag. **49**, 62 (2011)
12. E. Ip, J.M. Kahn, J. Lightwave Techn. **26**, 3416 (2008)
13. K. Toyoda, Y. Koizumi, T. Omiya, M. Yoshida, T. Hirooka, M. Nakazawa, Opt. Express **20**, 19815 (2012)
14. J. Hecht, New Scientist **213**, 19 (2012)
15. T. Morioka, paper FT 4 in: *Proc. OptoElectron. Commun. Conf. (OECC), Hong Kong* (2009)
16. S. Mushid, B. Grossman, P. Narakorn, Opt. Laser Technol. **40**, 1030 (2008)
17. J. Sakaguchi, B.J. Puttnam, W. Klaus, J.M.D. Mendinueta, Y. Awaji, N. Wada, A. Kanno, T. Kawanishi, Paper OW1I.3 in: *Tech. Digest of The Optical Fiber Communication Conference and Exposition (OFC) and the National Fiber Optic Engineers Conference (NFOEC) Anaheim* (2013)
18. D. Qian, et al., Paper FW6C.3 in: *Front. Opt. Conf. (FiO), Rochester*, 2012

19. F. Poletti, N.V. Wheeler, M.N. Petrovich, N. Baddela, E.N. Fokoua, J.R. Hayes, D.R. Gray, Z. Li, R. Slavik, D.J. Richardson, Nat. Photonics **7**, 279 (2013)
20. S. Matsuo, Y. Sasaki, T. Akamatsu, I. Ishida, K. Takenaga, K. Okuyama, K. Saitoh, M. Kosihba, Opt. Express **20**, 28398 (2012)
21. K. Saitoh, S. Matsuo, J. Lightw. Techn. DOI: 10.1109/JLT.2015.2466444 (2016)
22. J. Sakaguchi, W. Klaus, J.-M.D. Mendinueta, B.J. Puttnam, R.S. Luis, Y. Awaji, N. Wada, T. Hayashi, T. Nakanishi, T. Watanabe, Y. Kokubun, T. Takahata, T. Kobayashi, Postdeadline Paper Th5C.2 in: *Tech. Dig. Opt. Fiber Commun. Conf. Exposition (OFC) Los Angeles* (2015)
23. A. Hasegawa, F. Tappert, Appl. Phys. Lett. **23**, 142 (1973)
24. L.F. Mollenauer, R.H. Stolen, J.P. Gordon, Phys. Rev. Lett. **45**, 1095 (1980)
25. G. Agrawal, *Nonlinear Fiber Optics*, 5^{th} edn (Academic Press, San Diego, 2013)
26. L.F. Mollenauer, J.P. Gordon, *Solitons in Optical Fibers: Fundamentals and Applications* (Academic Press, San Diego, 2006)
27. F. Mitschke, *Fiber Optics. Physics and Technology* (Springer, Heidelberg, 2009)
28. N.N. Akhmediev, G. Town, S. Wabnitz, Opt. Comm. **104**, 385 (1994)
29. J.E. Prilepsky, et al., Phys. Rev. Lett. **108**, 183902 (2012)
30. O. Yushko, et al., J. Exper. Theor. Phys. **119**, 787 (2014)
31. J.H.B. Nijhof, N.J. Doran, W. Forysiak, F.M. Knox, Electron. Lett. **33**, 1726 (1997)
32. Y. Chen, H.A. Haus, Opt. Lett. **23**, 1013 (1998)
33. S.K. Turitsyn, E.G. Shapiro, Opt. Lett. **23**, 682 (1998)
34. J.N. Kutz, S.G. Evangelides, Opt. Lett. **23**, 685 (1998)
35. V.S. Grigoryan, C.R. Menyuk, Opt. Lett. **23**, 609 (1998)
36. N.J. Smith, F.M. Knox, N.J. Doran, K.J. Blow, I. Bennion, Electron. Lett. **32**, 54 (1996)
37. S. Turitsyn, E. Shapiro, S. Medvedev, M.P. Fedoruk, V. Mezentsev, C.R. Physique **4**, 145 (2003)
38. P.M. Lushnikov, J. Opt. Soc. Am. B **21**, 1913 (2004)
39. S.K. Turitsyn, B.G. Bale, M.P. Fedoruk, Phys. Rep. **521**, 135 (2012)
40. J.P. Gordon, Opt. Lett. **8**, 596 (1983)
41. F. Mitschke, L.F. Mollenauer, Opt. Lett. **12**, 355 (1987)
42. M. Stratmann, T. Pagel, F. Mitschke, Phys. Rev. Lett. **95**, 143902 (2005)
43. C. Paré, P.-A. Bélanger, Opt. Commun. **168**, 103 (1999)
44. A. Maruta, T. Inoue, Y. Nonaka, Y. Yoshika, IEEE J. Selected Top. Quant. El. **8**, 640 (2002)
45. M.J. Ablowitz, T. Hirooka, T. Inoue, JOSA B **19**, 2876 (2002)
46. B.-F. Feng, B.A. Malomed, Opt. Commun. **229**, 173 (2004)
47. S. Johnson, S. Pau, F. Küppers, J. Lightwave Technol. **29**, 3493 (2011)
48. F.J. Diaz-Otero, P. Chamorro-Posada, Opt. Commun. **285**, 162 (2012)
49. A. Hause, H. Hartwig, M. Böhm, F. Mitschke, Phys. Rev. A **78**, 063817 (2008)
50. A. Maruta, Y. Yoshika, Eur. Phys. J. Special Topics **173**, 139 (2009)
51. U. Al Khawaja, Phys. Rev. E **81**, 056603 (2010)
52. U. Al Khawaja, Abdelâali Boudjemâa, Phys. Rev. E **86**, 036606 (2012)
53. A. Boudjemâa, U. Al Khawaja, Phys. Rev. A **88**, 045801 (2013)
54. A. Hause, F. Mitschke, Phys. Rev. A **88**, 063843 (2013)
55. S.M. Alamoudi, U. Al Khawaja, B.B. Baizakov, Phys. Rev. A **89**, 053817 (2014)
56. A. Hause, H. Hartwig, B. Seifert, H. Stolz, M. Böhm, F. Mitschke, Phys. Rev. A **75**, 063836 (2007)
57. R. Trebino, *Frequency Resolved Optical Gating: The Measurement of Ultrashort Laser Pulses.* (Kluwer Academic Publishers, Boston, 2002)
58. B. Seifert, A. Hause, F. Mitschke, Appl. Phys. B **99**, 423 (2010)
59. P. Rohrmann, A. Hause, F. Mitschke, Sci. Rep. **2**, 866 (2012)
60. P. Rohrmann, A. Hause, F. Mitschke, Phys. Rev. A **87**, 043834 (2013)
61. A.M. Weiner, Rev. Scient. Instr. **71**, 1929 (2000)
62. S. Gholami, Ph. Rohrmann, A. Hause, F. Mitschke, Appl. Phys. B **116**, 43 (2013)